CALCULATION OF PRESSURE UNDER A THERMALLY STABLE SHELL IN A COMPOSITE COATING

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A method of determining the pressure under a thermally stable shell of a combined coating is examined when heated, and when a temperature, at which thermal destruction of the material takes place, is established and maintained on the surface of the thermally insulating substratum.

In design practice in various fields of technology we find that coatings are used which consist of a thin layer of thermally stable material, for example, a refractory metal, and a low heat-conducting substrate, which is a heat-insulating layer. Various types of resins, rubber, or similar organic compositions are usually used as the material of the substrate.

On high-temperature heating the thermally stable layer quickly heats up right through, and a constant temperature which is sufficient to destroy the structure of the material is established, and maintained on the surface of the substrate. Hence the material of the substrate begins to disintegrate with evolution of gaseous products of disintegration, and formation of a continuously increasing porous coke–gas layer. If the surface layer is hermetically sealed, and removal of disintegration products is not guaranteed, a pressure increase takes place under the layer of thermally stable material; in some cases it is desirable to know the magnitude of this pressure increase.

On examining a given problem we will make the following assumptions:

1) as new portions of gaseous products of disintegration are evolved, as a result of displacement of the disintegration zone, the pressure in the coke–gas layer is momentarily equalized;
2) at each point of the disintegrating layer, as the temperature increases, the material slowly gasifies to its limiting value at a given temperature.

As a rule the degree of full gasification in relation to the temperature for each material is determined experimentally. For coking materials disintegration is usually completed at temperatures of ~1000°C.

On the basis of the second assumption for a selected moment of time, the mass of gas evolved in the column of the coke–gas layer on a unit area of the lateral cross section can be determined as

$$M_{\mathrm{g}}=\int_{0}^{x_2}\rho_2\Gamma dx. \tag{1}$$

On the other hand, the indicated mass of gas

$$M_{\mathrm{g}}=\int_{0}^{x_2}\rho_{\mathrm{g}}f dx. \tag{2}$$

If we consider that the gas is subject to the equation of state, and the porosity is a function of the current value of the density of the material

$$f=f_{\mathrm{fc}}\frac{\rho-\rho_2}{\rho_{\mathrm{fc}}-\rho_2}\,, \tag{3}$$

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expression (2) can be rewritten as follows:

$$M_{\mathrm{g}} = \int\limits_0^{x_2} \frac{Pf_{\mathrm{fc}}(\rho_2 - \rho)}{RT_1(\rho_2 - \rho_{\mathrm{fc}})}\, dx = \int\limits_0^{x_2} \frac{Pf_{\mathrm{fc}}\rho_2 G}{RT_1(\rho_2 - \rho_{\mathrm{fc}})}\, dx. \tag{4}$$

The current density of the nondisintegrating material is associated with the degree of gasification by the ratio $\rho = \rho_2(1 - G)$. We compare expressions (1) and (4), transferring from the integration sign magnitudes which are not functions of X, and we solve them for P. Hence we obtain a relationship, according to which it is possible to determine the magnitude of the pressure in the disintegrating material:

$$P = \frac{R(\rho_2 - \rho_{\mathrm{fc}}) \int\limits_0^{x_2} G\, dx}{f_{\mathrm{n}} \int\limits_0^{x_2} \frac{G}{T_1}\, dx}, \tag{5}$$

hence it is seen that the magnitude of the pressure as a whole is determined by the temperature field in a porous coke–gas layer, since the degree of gasification of the material depends only on the temperature. The temperature field can be determined by analytical solution of the combined problem of thermal conductivity, which is formulated as follows:

in the layer of the original material

$$\frac{\partial T_2}{\partial t} = a_2 \frac{\partial^2 T_2}{\partial x^2}; \quad a_2 = \frac{\lambda_2}{c_2 \rho_2} \quad \text{for} \quad x_2(t) < x < \infty, \tag{6}$$

in the coke–gas layer

$$(\rho c_p)_\Sigma \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_\Sigma \frac{\partial T}{\partial x}\right) \quad \text{for} \quad 0 < x < x_2(t). \tag{7}$$

Hence the effective values of heat conductivity, and the specific heat of the coked layer can be represented, for example, by the following relationships [2]:

$$\begin{gathered} \lambda_\Sigma = f\lambda_{\mathrm{g}} + (1 - f)\lambda_{\mathrm{co}}, \\ (\rho c_p)_\Sigma = f(\rho c_p)_{\mathrm{g}} + (1 - f)\rho_{\mathrm{co}} c_{\mathrm{co}}. \end{gathered} \tag{8}$$

Equation (7) is written on the assumption that since there is no pressure gradient over the thickness of the coke–gas layer, the process of heat transfer takes place only as a result of thermal conductivity.

Porosity in a general case is a magnitude which is variable over the thickness of the coke layer; however, for simplicity in calculating the temperature field we can assume that it is a constant value equal to the porosity of a fully coked layer, and we consider λ_Σ and a_Σ to be constant values in the given region, as a result of their negligible variation over the thickness.

Taking the above mentioned observation into account, Eq. (7) will be transcribed as follows:

$$\frac{\partial T_1}{\partial t} = a_\Sigma \frac{\partial^2 T_1}{\partial x^2}; \quad a_\Sigma = \frac{\lambda_\Sigma}{(\rho c_p)_\Sigma} \quad \text{for} \quad 0 < x < x_2(t). \tag{7'}$$

The system of Eqs. (6), and (7') is solved for the following boundary, and initial conditions:

$$T_2(x, 0) = T_0, \quad T_1(0, t) = T_w, \tag{9}$$

$$T_1 = T_2 = T_m \quad \text{for} \quad x = x_2(t), \tag{10}$$

$$\frac{\partial T_2(\infty, t)}{\partial x} = 0, \tag{11}$$

$$\left(-\lambda_\Sigma \frac{\partial T_1}{\partial x} + \lambda_2 \frac{\partial T_2}{\partial x}\right)_{x = x_2} = \rho_2 L \frac{dx_2}{dt}. \tag{12}$$

The condition (12) is approximate since, when solving the thermal problem, it is assumed that the irreversible reactions of thermal disintegration, accompanied by thermal effects, take place only on a moving boundary which is determined by the function $x_2(t)$. As a result we came to the known Stefan problem for which a general solution is given in [1].

In order to determine the position of the disintegration front and the distribution of temperatures in the coke–gas layer we have the relationships

$$x_2 = \beta \sqrt{t},$$

$$T_1(x,\ t) = T_w + (T_m - T_w)\frac{\operatorname{erf}\dfrac{x}{2\sqrt{a_\Sigma t}}}{\operatorname{erf}\dfrac{\beta}{2\sqrt{a_\Sigma}}}.$$

β is determined from the characteristic equation

$$\frac{\lambda_\Sigma (T_w - T_m)}{\sqrt{a_\Sigma}\operatorname{erf}\dfrac{\beta}{2\sqrt{a_\Sigma}}}\exp\left(-\frac{\beta^2}{4a_\Sigma}\right) + \frac{\lambda_2 (T_m - T_0)}{\sqrt{a_2}\operatorname{erfc}\dfrac{\beta}{2\sqrt{a_2}}}\exp\left(-\frac{\beta^2}{4a_2}\right) = \frac{\rho_2 L\sqrt{\pi}}{2}\beta.$$

Having hence determined the position of the disintegration front $x_2(t)$, and the profile of temperatures in the coke–gas layer $T_1(x, t)$, it is possible to determine the magnitude of the pressure under the thermally stable shell of the composite coating for a selected moment of time, according to expression (5).

NOTATION

x	is the coordinate from the internal surface of the thermally stable layer;
ρ	is the density;
T	is the temperature;
G	is the degree of gasification;
f	is the porosity;
P	is the pressure;
R	is the gas constant;
λ	is the coefficient of heat conductivity;
c	is the specific heat;
L	is the thermal effect of disintegration reaction;
t	is the time;
$x_2(t)$	is the position of the front of the beginning of disintegration.

Subscripts

1	are the conditions in the zone of the material which is undergoing destruction;
2	are the conditions in the zone of nondisintegrated material;
co	refers to material of coke residue;
g	refers to gaseous disintegration products;
fc	refers to magnitudes characterizing the fully coked residue.

LITERATURE CITED

1. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Izd. Vysshaya Shkola (1967).
2. S. M. Scala and J. M. Gilbert, "Thermal degradation of charring plastics during hypersonic flight," ARS Journal, 32, No. 6 (1962).